

Improved quark mass density- dependent model with quark-sigma meson and quark-omega meson couplings

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Abstract

An improved quark mass density- dependent model with the non-linear scalar sigma field and the ω -meson field is presented. We show that the present model can describe saturation properties, the equation of state, the compressibility and the effective nuclear mass of nuclear matter under mean field approximation successfully. The comparison of the present model and the quark-meson coupling model is addressed.

PACS numbers: 12.39.-x; 14.20.-c; 05.45.Yv

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I. INTRODUCTION

Owing to the non-perturbative nature of QCD in low energy regions, phenomenological models reflecting the characteristic of the strong interaction are widely used in the studying of the properties of hadrons, nuclear matter and quark matters[1-12]. They are based on different degrees of freedom, for example: nucleons and mesons, or quarks and gluons, in particular, on hybrid quarks and mesons. Some of these models have been proved to be successful. The quark mass density-dependent model(QMDD)[5] is one of such candidates.

According to the QMDD model, the masses of u, d quarks and strange quarks (and the corresponding anti-quarks) are given by

$$m_q = \frac{B}{3n_B} (i = u, d, \bar{u}, \bar{d}) \quad (1)$$

$$m_{s,\bar{s}} = m_{s0} + \frac{B}{3n_B} \quad (2)$$

where n_B is the baryon number density, m_{s0} is the current mass of the strange quark, and B is the bag constant. At zero temperature

$$n_B = \frac{1}{3}(n_u + n_d + n_s), \quad (3)$$

where n_u, n_d, n_s represent the density of u quark, d quark, and s quark, respectively. The basic hypothesis Eqs.(1) and (2) corresponds to a quark confinement mechanism because if quark goes to infinite space, the volume of the system tends to infinite, n_B approaches to zero and the m_q goes to infinite, and the infinite quark mass prevents the quark from going to infinite. The confinement mechanism is similar to that of the MIT bag model.

Although the QMDD model can provide a description of confinement and explain many dynamical properties of strange quark matter, but it is still an ideal quark gas model and cannot explain the temperature T vs. density ρ deconfinement phase diagram of QCD and the properties of nuclear matter[13-14]. To overcome this difficulty, we have introduced a coupling between quark and nonlinear scalar field to improve the QMDD model in a previous paper[15]. We have found the wave functions of the ground state and the lowest one-particle excited states. By using these wave functions, we calculated many physical quantities such as root-mean-squared radius, the magnetic moment of nucleon to compare with experiments and come to a conclusion that this improved QMDD model is successful to explain the

properties of nucleon. In ref[16], we extended this model to finite temperature and studied its soliton solution by means of the finite temperature quantum field theory. The critical temperature of quark deconfinement T_C and the temperature-dependent bag constant $B(T)$ are found. The results of improved QMDD(IQMDD) model are qualitatively similar to that obtained from Freidberg-Lee soliton bag model[17, 18].

Instead of studying the nucleon properties, we hope to employ the IQMDD to investigate the physical properties of nuclear matter in this paper. As was shown by the Walecka model[1] and the QMC model[7-10] early, a neutral vector field coupled to the conserved baryon current is very important for describing bulk properties of nuclear matter. The large neutral scalar and vector contributions have been observed empirically from NN scattering amplitude. The main qualitative features of the nucleon-nucleon interaction: a short range repulsion between baryons coming from ω -meson exchange, and a long-range attraction between baryons coming from σ -meson exchange must be included in a successful model. Obviously, if we hope to employ the IQMDD model to mimick this repulsive and attractive interactions, except the quark and σ -meson interaction, the ω meson and the $qq\omega$ coupling must be added. This motivate us to introduce ω mesons and the $qq\omega$ coupling in the IQMDD model in this paper. In this new IQMDD model, the nonlinear scalar field coupling with quarks forms a soliton bag, and the $qq\omega$ vector coupling gives the repulsion between quarks. We will prove that this model can give us a successful description of nuclear matter.

The organization of this paper is as follows. In the next section, we give the main formulae of the IQMDD model under the mean field approximation at zero temperature. In the third section, some numerical results are contained. The last section contains a summary and discussions.

II. FORMULAE OF THE IMPROVED QMDD MODEL

The Lagrangian density of the IQMDD model is :

$$L = \bar{\psi}[i\gamma^\mu\partial_\mu - m_q + g_\sigma^q\sigma - g_\omega^q\gamma^\mu\omega_\mu]\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \quad (4)$$

where

$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \quad (5)$$

and the quark mass m_q is given by Eqs.(1) and (2), m_σ and m_ω are the masses of σ and ω mesons, g_σ^q and g_ω^q are the couplings constant between quark- σ meson and quark- ω meson respectively. And

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}b\sigma^3 + \frac{1}{4}c\sigma^4 + B \quad (6)$$

$$-B = \frac{m_\sigma^2}{2}\sigma_v^2 + \frac{b}{3}\sigma_v^3 + \frac{c}{4}\sigma_v^4. \quad (7)$$

where σ_v is the absolute minimum of $U(\sigma)$, $U(\sigma_v) = 0$ and $U(0) = B$.

It can easily show the equation of motion for quark field in the whole space is

$$[\gamma^\mu(i\partial_\mu + g_\omega^q\omega_\mu) - (m_q - g_\sigma^q\sigma)]\psi = 0 \quad (8)$$

Under mean field approximation, the effective quark mass m_q^* is given by:

$$m_q^* = m_q - g_\sigma^q\bar{\sigma} \quad (9)$$

In nuclear matter, three quarks constitute a bag, and the effective nucleon mass is obtained from the bag energy and reads:

$$M_N^* = \Sigma_q E_q = \Sigma_q \frac{4}{3}\pi R^3 \frac{\gamma_q}{(2\pi)^3} \int_0^{K_F^q} \sqrt{m_q^{*2} + k^2} \left(\frac{dN_q}{dk}\right) dk \quad (10)$$

where quark degeneracy $\gamma_q=6$, K_F^q is the Fermi energy of quarks. dN_q/dk is the density of states for various quarks in a spherical cavity. It is given by[19]:

$$N(k) = A(kR)^3 + B(KR)^2 + C(KR) \quad (11)$$

where

$$A = \frac{2\gamma_q}{9\pi}. \quad (12)$$

$$B\left(\frac{m_q}{k}\right) = \frac{\gamma_q}{2\pi} \left\{ \left[1 + \left(\frac{m_q}{k}\right)^2\right] \arctan\left(\frac{k}{m_q}\right) - \frac{m_q}{k} - \frac{\pi}{2} \right\}. \quad (13)$$

$$C\left(\frac{m_q}{k}\right) = \tilde{C}\left(\frac{m_q}{k}\right) + \left(\frac{m_q}{k}\right)^{1.45} \frac{\gamma_q}{3.42\left(\frac{m_q}{k} - 6.5\right)^2 + 100}. \quad (14)$$

$$\tilde{C}\left(\frac{m_q}{k}\right) = \frac{\gamma_q}{2\pi} \left\{ \frac{1}{3} + \left(\frac{m_q}{k} + \frac{k}{m_q}\right) \arctan\left(\frac{k}{m_q}\right) - \frac{\pi k}{2m_q} \right\}. \quad (15)$$

Eqs. (12) and (13) are in good agreement with those given by multireflection theory[21, 22] and the Eqs. (14) and (15) are given by a best fit of numerical calculation for the MIT bag model. The curvature term \tilde{C} cannot be evaluated by this theory except for two limiting cases $m_q \rightarrow 0$ and $m_q \rightarrow \infty$. Madsen[20] proposed the Eq. (15), but as was pointed out by Ref. [19], the best fit of numerical data is given by Eq. (14). This density of state has been employed by Refs. [13,14] to study the strangelets.

The Fermi energy K_F^q of quarks is given by

$$3 = \frac{4}{3}\pi R^3 n_B \quad (16)$$

where n_B satisfies

$$n_B = \Sigma_q \frac{\gamma_q}{(2\pi)^3} \int_0^{K_F^q} \left(\frac{dN_q}{dk}\right) dk \quad (17)$$

The bag radius R is determined by the equilibrium condition for the nucleon bag:

$$\frac{\delta M_N^*}{\delta R} = 0 \quad (18)$$

In nuclear matter, the total energy density is given by

$$\varepsilon_{matter} = \frac{\gamma_N}{(2\pi)^3} \int_0^{K_F^N} \sqrt{M_N^{*2} + p^2} dp^3 + \frac{g_\omega^2}{2m_\omega^2} \rho_B^2 + \frac{1}{2} m_\sigma^2 \bar{\sigma}^2 + \frac{1}{3} b \bar{\sigma}^3 + \frac{1}{4} c \bar{\sigma}^4 \quad (19)$$

where $\gamma_N = 4$ is degeneracy of nucleon, K_F^N is fermi energy of nucleon and ρ_B is the density of nuclear matter

$$\rho_B = \frac{\gamma_N}{(2\pi)^3} \int_0^{K_F^N} d^3 k \quad (20)$$

In Eqs. (19), g_ω is the coupling constant between the nucleon and the ω meson and it satisfies $g_\omega = 3g_\omega^q$. As that of the QMC model[7], the $\bar{\sigma}$ is yielded by the equation:

$$m_\sigma^2 \bar{\sigma} + b \bar{\sigma}^2 + c \bar{\sigma}^3 = -\frac{\gamma_N}{(2\pi)^3} \int_0^{K_F^N} \frac{M_N^*}{\sqrt{M_N^{*2} + p^2}} d^3 p \left(\frac{\partial M_N^*}{\partial \bar{\sigma}}\right)_R \quad (21)$$

Eqs. (9)-(21) form a complete set of equations and we can solve them numerically. Our numerical results will be shown in the next section.

III. NUMERICAL RESULT

Before numerical calculation, let us consider the parameters in IQMDD model. As that of Ref.[1, 23], the masses of ω -meson and σ -meson are fixed as $m_\omega = 783$ MeV, $m_\sigma = 509$

MeV respectively. We choose the bag constant $B = 174 \text{ MeV fm}^{-3}$ to fit the mass of nucleon $M_N = 939 \text{ MeV}$. When B is determined, the parameters b and c are not independent because of Eq. (7). we choose the b is free parameter. There are still three parameters, namely, $g_\omega^q, g_\sigma^q, b$ are needed to be fixed in IQMDD model.

To study the physical properties of nuclear matter, we investigate the nuclear saturation, the equation of state and the compressibility. The pressure of nuclear matter P is given by

$$P = \rho_B^2 \frac{\partial}{\partial \rho_B} \frac{\varepsilon_{matter}}{\rho_B} \quad (22)$$

where ρ_B is the baryon density. The compressibility for nuclear matter reads:

$$K = 9 \frac{\partial}{\partial \rho_B} P \quad (23)$$

at saturation point, the binding energy per particle $E/A = -15 \text{ MeV}$, and the saturation density $\rho_0 = 0.15 \text{ fm}^{-3}$.

Our numerical results are shown in Fig. 1-Fig. 4. In Fig. 1, we choose ω -meson and σ -meson satisfy $\bar{\sigma} = 0, \bar{\omega} = 0$ and depict the bag energy as a function of bag radius at zero temperature. We find the stable radius of a "free" nucleon $R = 0.85 \text{ fm}$.

In Figs. 2-4 we show the effective mass M^* of nucleon, the saturation curve and the equations of state of nuclear matter at zero temperature for IQMDD model respectively, where we fix the parameter $b = -1460 \text{ (MeV)}$, $g_\sigma = 4.67$ and $g_\omega = 2.44$ respectively, and find $E/A = -15 \text{ MeV}$ and $\rho_0 = 0.15 \text{ fm}^{-3}$ and $K(\rho_0) = 210 \text{ MeV}$. We find our model can explain the properties of nuclear matter successfully.

To illustrate our results more transparently, we show the dependence of the properties of nuclear matter on the parameters $b, g_\sigma^q, g_\omega^q$ in Table.1 for fixing binding energy $E/A = -15 \text{ MeV}$ and $\rho_0 = 0.15 \text{ fm}^{-3}$. We find that the compressibility $K(\rho_0)$ and effective nucleon mass $M_N^*(\rho_0)$ at saturation point all decrease when g_σ^q, g_ω^q increase and b decreases. As was shown in Table.1, the variational regions for $K(\rho_0)$ and $M_N^*(\rho_0)$ are small, and the decreasements of $K(\rho_0)$ and $M_N^*(\rho_0)$ are slowly.

TABLE 1. Variation of the nuclear matter properties to b .

$b(\text{MeV})$	g_σ^q	g_ω^q	$K(\rho_0)(\text{MeV})$	$M_N^*(\rho_0)(\text{MeV})$
-800	4.59	2.35	218.8	782.9
-1000	4.61	2.38	215.5	781.2
-1200	4.64	2.40	213.6	778.5
-1400	4.66	2.43	211.2	776.8
-1600	4.69	2.46	208.1	774.3
-1800	4.71	2.48	205.7	772.6

It was pointed in the Refs. [24] early, adding a nonlinear scalar field in the model will cause unphysical behavior under mean field approximation in nuclear matter. This can easily be seen from Eq.(21) because the left hand side of Eq. (21) is a cubic order function of $\bar{\sigma}$, and $\bar{\sigma} = 0$ is one of its solutions. There are two solutions in low-density regions. In Fig. 5 and Fig. 6, these two solutions are shown explicitly for $\bar{\sigma}$ vs. ρ_B curve and for M_N^* vs. ρ_B curve respectively where the parameters are fixing as $b = -3655$ (MeV). Noting that the term of non-linear scalar field is essential to form a soliton bag, we conclude that the unphysical branch cannot be avoided for the soliton solution under mean field approximation. Fortunately, the lower branch cannot be ended at the point($M_N = 939$ MeV, $\rho_B = 0$), and give us a experimental value of nucleon mass, we will give up this unphysical branch in our calculation.

Finally, It is of interest to compare the properties of nuclear matter for IQMDD model and for the QMC model. Our results are shown in Table. 2. we find their results are very similar. But as was pointed in our previous paper[15], the first advantage of the IQMDD model is that the MIT bag boundary constraint has been given up because it mimicks to a Friedberg-lee soliton bag model[13-16]. The second advantage of the IQMDD model is that the interactions between $qq\sigma$ and $qq\omega$ are extended to the whole free space. We can easily write down the propagators of quarks, σ -meson and ω -meson respectively and do the many-body calculations beyond mean field approximation. But for QMC model, the propagators of quarks, σ -meson and ω -meson cannot be written down easily because one must consider the multireflection by the MIT bag boundary as well as the effect of the interactions limited into a nucleon space only. The IQMDD model provides a good substitute of the QMC model which is more suitable for the study of nuclear matter beyond

mean field.

TABLE 2. Comparison of properties for IQMDD and QMC model.

	R(fm)	g_σ^q	g_ω^q	$K(\rho_0)$ (MeV)	$M_N^*(\rho_0)$ (MeV)
QMC	0.80	5.53	1.26	200	851
IQMDD(b=0)	0.85	4.54	2.21	227	798
IQMDD(b=-1460)	0.85	4.67	2.44	210	775
QHD-1		$g_\sigma = 9.57$	$g_\omega = 11.6$	540	522

IV. SUMMARY AND DISCUSSION

In summary, we present an Improved quark mass density dependent model which has the non-linear σ meson field, and the ω meson field. The $qq\sigma$ coupling and the $qq\omega$ coupling are introduced to mimic the repulsive and the attractive interactions between quarks in this model. It is shown that the present model is successful for describing the saturation properties, the equation of state and compressibility of nuclear matter. The effective nucleon mass decreases with baryon density in this model more rapidly than that of QMC model. After comparing the IQMDD model and the QMC model, we come to a conclusion that the IQMDD model is a good substitute for QMC model.

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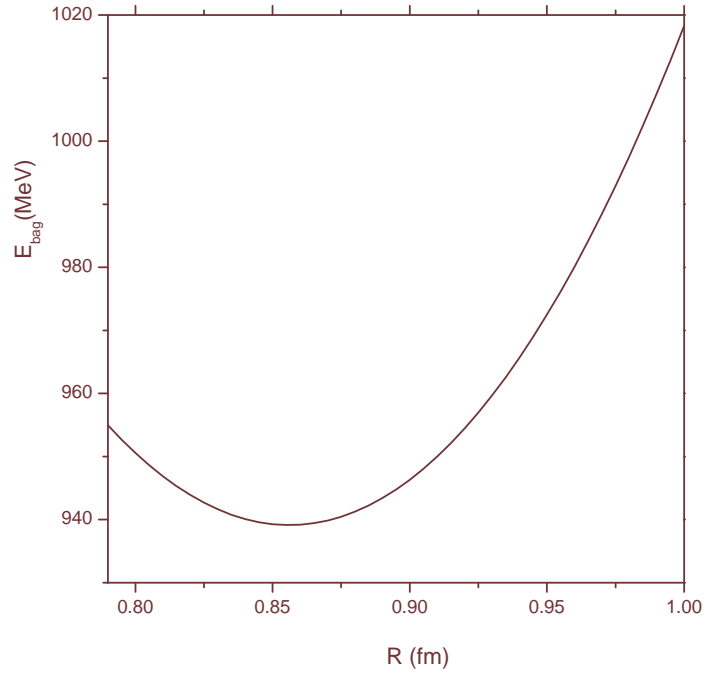


Fig.1

FIG. 1: The bag energy as a function of bag radius at zero temperature for $\bar{\omega} = 0, \bar{\omega} = 0$.

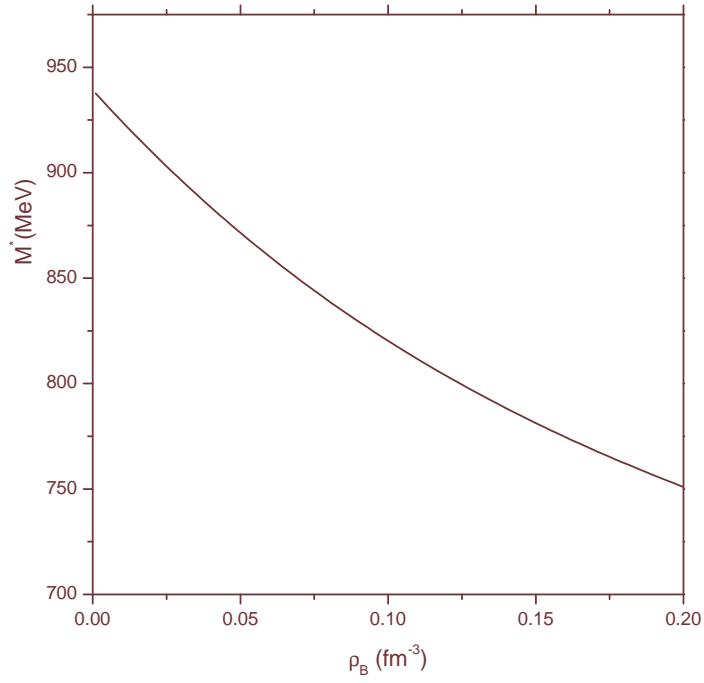


Fig.2

FIG. 2: Effective nucleon mass vs. baryon density at zero temperature where the parameters $g_\sigma^q = 4.67, g_\omega^q = 2.44, b = -1460$ (MeV).

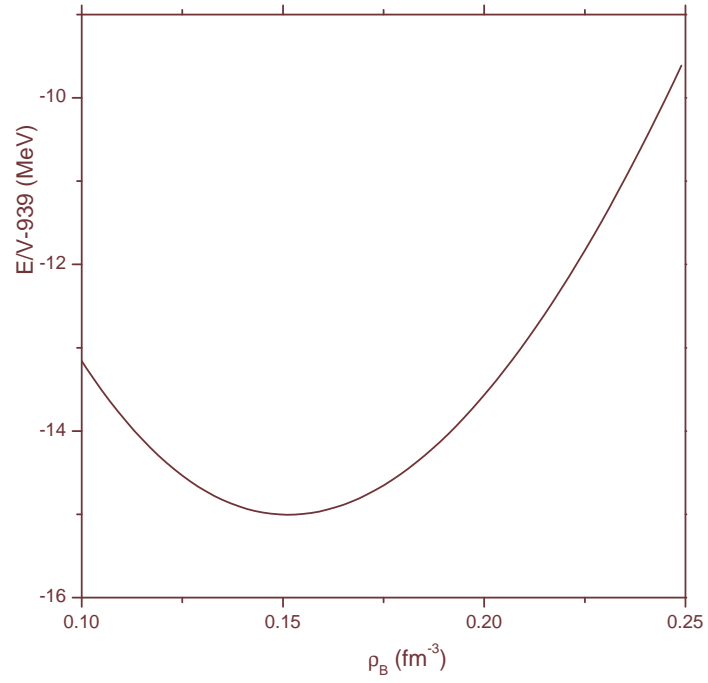


Fig.3

FIG. 3: Saturation curve of nuclear matter at zero temperature. the parameters is same as that of Fig. 2.

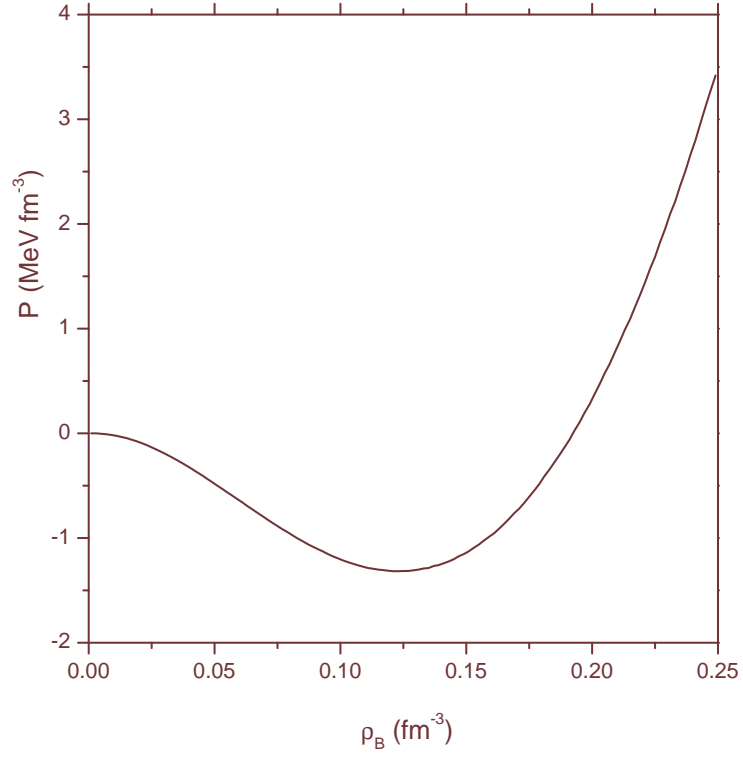


Fig.4

FIG. 4: Pressure of nuclear matter as a function of ρ_B . the parameters is same as that of Fig. 2.

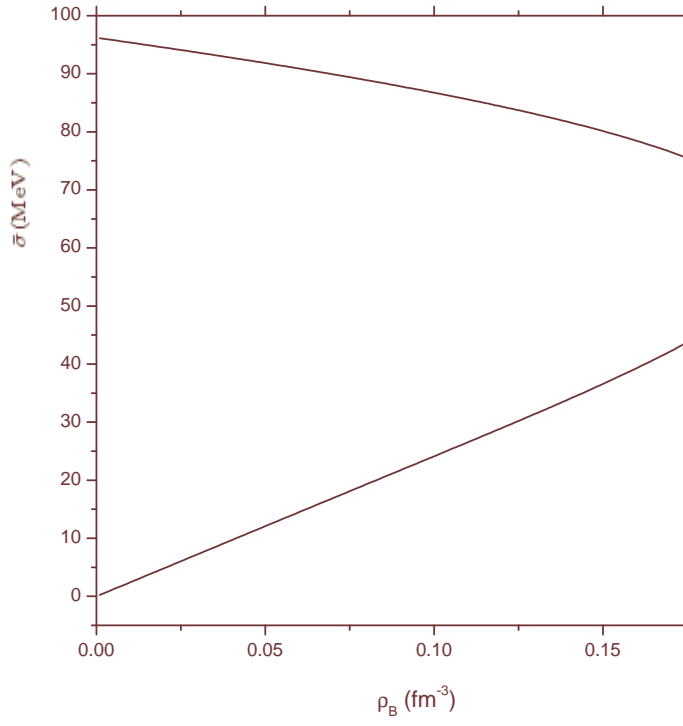


Fig.5

FIG. 5: the $\bar{\sigma}$ field vs. baryon density for $b=-3655$ (MeV), $g_\sigma^q = 5.23, g_\omega^q = 3.12$.

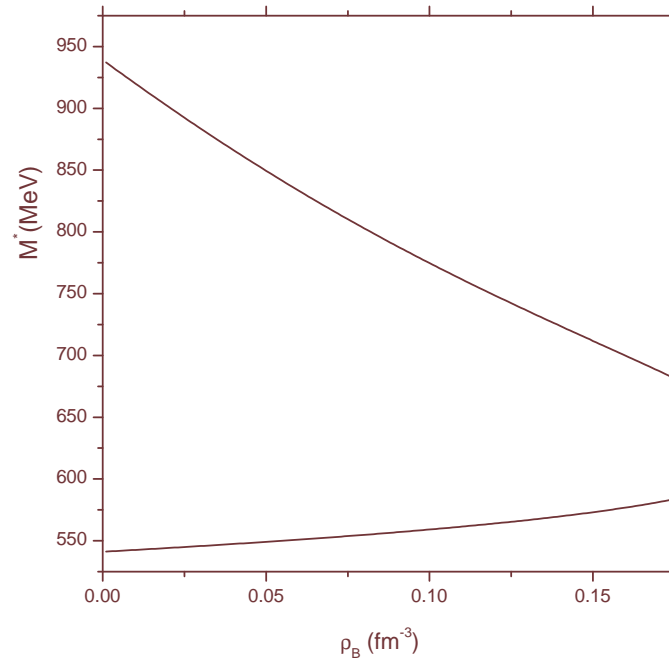


Fig.6

FIG. 6: Effective nucleon mass M^* vs. baryon density. the parameters is same as that of Fig. 5.